Decision Networks

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Lecture 12

Readings: RN 16.5 - 16.6. PM 9.1 - 9.4.

Outline

Decision Theory

How should an agent act in an uncertain world?

- What should the agent believe based on the evidence?
- What does the agent want?

  $\rightarrow$ Associate each state of the world with a real number.

The principle of maximum expected utility (MEU):

A rational agent should choose the action that maximizes the agent's expected utility.

Decision Networks

Decision networks
$= \textit{Bayesian network} + \textit{actions} + \textit{utilities}$

Running example: a mail pick-up robot

The robot must choose its route to pick up the mail. There is a short route and a long route. On the short route, the robot might slip and fall. The robot can put on pads. Pads won't change the probability of an accident. However, if an accident happens, pads will reduce the damage. Unfortunately, the pads add weight and slow the robot down. The robot would like to pick up the mail as quickly as possible while minimizing the damage caused by an accident.

What should the robot do?

Variables

What are the random variables?

$\rightarrow$ A: whether an accident occurs or not.

What are the decision variables (actions)?

$\rightarrow$ P: whether the robot puts on pads.

S: whether the robot chooses the short route.

Nodes in a Decision Network

- Chance nodes

  represent random variables (as in Bayesian networks).

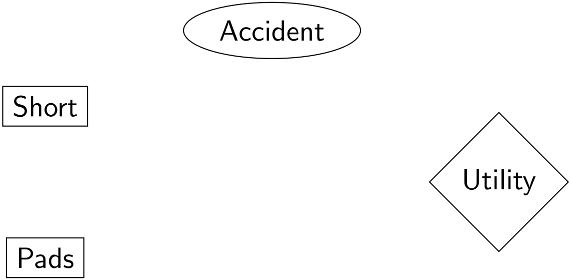
- Decision nodes

  represent actions (decision variables).

- Utility node

  represents agent's utility function on states (happiness in each state).

Robot decision network

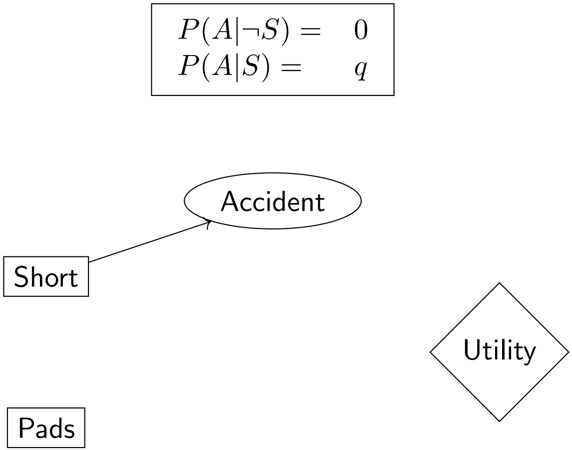


Arcs in the Decision Network

How do the random variables and the decision variables relate to one another?

- *Short* affects *Accident*. If the robot chooses the short route, an accident may occur. If the robot chooses the long route, an accident won't occur.
- *Pads* does not affect *Accident*.

Robot decision network



Question: The robot's happiness

Q #1: Which variables directly influence the robot's happiness?

(A) P only
(B) S only
(C) A only
(D) Two of (A), (B), and (C)
(E) All of (A), (B) and (C)

The robot must choose its route to pick up the mail. There is a short route and a long route. On the short route, the robot might slip and fall. The robot can put on pads. Pads won't change the probability of an accident. However, if an accident happens, pads will reduce the damage. Unfortunately, the pads add weight and slow the robot down. The robot would like to pick up the mail as quickly as possible while minimizing the damage caused by an accident.

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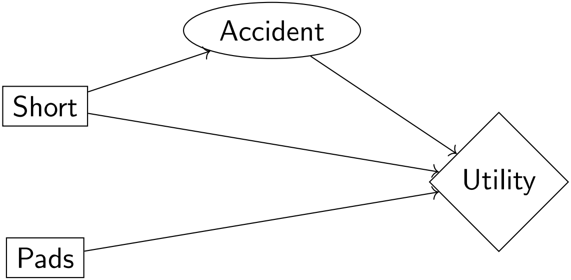
(A) P only
(B) S only
(C) A only
(D) Two of (A), (B), and (C)
(E) All of (A), (B) and (C)

→ (E) is correct.

Robot decision network

$$\begin{aligned} P(A|\neg S) &= 0 \\ P(A|S) &= q \end{aligned}$$



Question: The robot's utility function

Q #2: When an accident does NOT happen, which of the following is true?

(A) The robot prefers not wearing pads than wearing pads.

(B) The robot prefers the long route over the short route.

(C) Both (A) and (B) are true.

(D) Both (A) and (B) are false.

Question: The robot's utility function

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(A) The robot prefers not wearing pads than wearing pads.

(B) The robot prefers the long route over the short route.

(C) Both (A) and (B) are true.

(D) Both (A) and (B) are false.

→ (A) is correct.

The robot's utility function

	State	$U(w_i)$
$\neg P, \neg S, \neg A$	w_0 slow, no weight	6
$\neg P, \neg S, A$	w_1 impossible	
$\neg P, S, \neg A$	w_2 quick, no weight	10
$\neg P, S, A$	w_3 severe damage	0
$P, \neg S, \neg A$	w_4 slow, extra weight	4
$P, \neg S, A$	w_5 impossible	
$P, S, \neg A$	w_6 quick, extra weight	8
P, S, A	w_7 moderate damage	2

The robot's utility function

How does the robot's utility/happiness depend on the random variables and the decision variables?

- When an accident does not happen, does the robot prefer not wearing pads or wearing pads?

The robot's utility function

How does the robot's utility/happiness depend on the random variables and the decision variables?

- When an accident does not happen, does the robot prefer not wearing pads or wearing pads?

  → The robot prefers not wearing pads because it can move faster.

$$U(\neg P \land \neg S \land \neg A) > U(P \land \neg S \land \neg A)$$
$$U(\neg P \land S \land \neg A) > U(P \land S \land \neg A)$$

The robot's utility function

How does the robot's utility/happiness depend on the random variables and the decision variables?

- When an accident does not happen, does the robot prefer not wearing pads or wearing pads?

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$$U(\neg P \wedge \neg S \wedge \neg A) > U(P \wedge \neg S \wedge \neg A)$$
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- When an accident does not happen, does the robot prefer the short route or the long route?

  → The robot prefers the short route because it can be faster.

$$U(P \wedge S \wedge \neg A) > U(P \wedge \neg S \wedge \neg A)$$
$$U(\neg P \wedge S \wedge \neg A) > U(\neg P \wedge \neg S \wedge \neg A)$$

The robot's utility function

How does the robot's utility/happiness depend on the random variables and the decision variables?

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The robot's utility function

How does the robot's utility/happiness depend on the random variables and the decision variables?

- When an accident occurs, does the robot prefer the short route or the long route?

  $\rightarrow$ The robot must have taken the short route.
  Thus, there is no utility for $\neg P \wedge \neg S \wedge A$ and $P \wedge \neg S \wedge A$.

The robot's utility function

How does the robot's utility/happiness depend on the random variables and the decision variables?

- When an accident occurs, does the robot prefer the short route or the long route?

  $\rightarrow$ The robot must have taken the short route.
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- When an accident occurs, does the robot prefer not wearing pads or wearing pads?

The robot's utility function

How does the robot's utility/happiness depend on the random variables and the decision variables?

- When an accident occurs, does the robot prefer the short route or the long route?

  → The robot must have taken the short route.
  Thus, there is no utility for $\neg P \land \neg S \land A$ and $P \land \neg S \land A$.

- When an accident occurs, does the robot prefer not wearing pads or wearing pads?

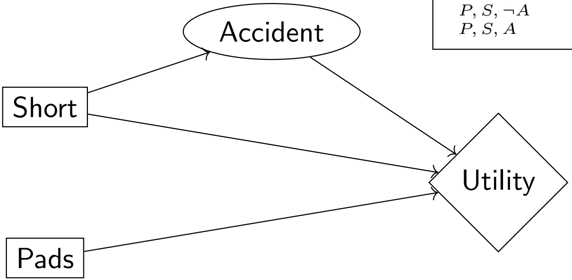
  → In this case, the robot prefers wearing pads than not wearing pads because pads reduce the severity of damage.

$$U(P \land S \land A) > U(\neg P \land S \land A)$$

Robot decision network

$$P(A|\neg S) = 0$$
$$P(A|S) = q$$

	State	$U(w_i)$
$\neg P, \neg S, \neg A$	w_0 slow, no weight	6
$\neg P, \neg S, A$	w_1 impossible	
$\neg P, S, \neg A$	w_2 quick, no weight	10
$\neg P, S, A$	w_3 severe damage	0
$P, \neg S, \neg A$	w_4 slow, extra weight	4
$P, \neg S, A$	w_5 impossible	
$P, S, \neg A$	w_6 quick, extra weight	8
P, S, A	w_7 moderate damage	2



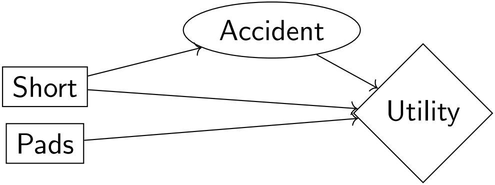
Evaluating a decision network

How do we choose an action?

1. Set evidence variables for current state
2. For each possible value of decision node
   (a) set decision node to that value
   (b) calculate posterior probability for parent nodes of the utility node
   (c) calculate expected utility for the action
3. Return action with highest expected utility

Calculating the expected utilities (1/4)

What is the agent's expected utility of not wearing pads and choosing the long route?

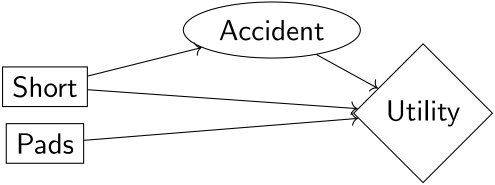


$\rightarrow$

$$
\begin{aligned}
EU(\neg P, \neg S) =& P(w_0|\neg P \wedge \neg S) * U(w_0) + P(w_1|\neg P \wedge \neg S) * U(w_1) \\
&= P(\neg P \wedge \neg S \wedge \neg A|\neg P \wedge \neg S) * U(w_0) \\
&\quad + P(\neg P \wedge \neg S \wedge A|\neg P \wedge \neg S) * U(w_1) \\
&= P(\neg A|\neg P \wedge \neg S) * U(w_0) + P(A|\neg P \wedge \neg S) * U(w_1) \\
&= P(\neg A|\neg S) * U(w_0) + P(A|\neg S) * U(w_1) \\
&= (1)(6) + (0)(-) \\
&= 6
\end{aligned}
$$

Calculating the expected utilities (2/4)

What is the agent's expected utility of not wearing pads and choosing the short route?

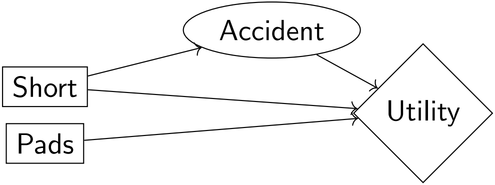


$\rightarrow$

$$
\begin{aligned}
EU(\neg P, S) &= P(w_2|\neg P \wedge S) * U(w_2) + P(w_3|\neg P \wedge S) * U(w_3) \\
&= P(\neg P \wedge S \wedge \neg A|\neg P \wedge S) * U(w_2) \\
&\quad + P(\neg P \wedge S \wedge A|\neg P \wedge S) * U(w_3) \\
&= P(\neg A|\neg P \wedge S) * U(w_2) + P(A|\neg P \wedge S) * U(w_3) \\
&= P(\neg A|S) * U(w_2) + P(A|S) * U(w_3) \\
&= (1-q)(10) + (q)(0) \\
&= 10 - 10q
\end{aligned}
$$

Calculating the expected utilities (3/4)

What is the agent's expected utility of wearing pads and choosing the long route?

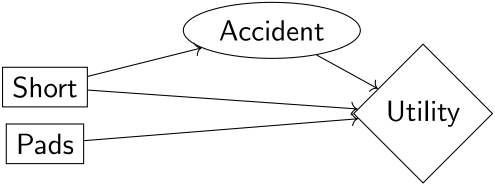


$\rightarrow$

$$
\begin{aligned}
EU(P, \neg S) &= P(w_4 | P \land \neg S) * U(w_4) + P(w_5 | P \land \neg S) * U(w_5) \\
&= P(P \land \neg S \land \neg A | P \land \neg S) * U(w_4) \\
&\quad + P(P \land \neg S \land A | P \land \neg S) * U(w_5) \\
&= P(\neg A | P \land \neg S) * U(w_4) + P(A | P \land \neg S) * U(w_5) \\
&= P(\neg A | \neg S) * U(w_4) + P(A | \neg S) * U(w_5) \\
&= (1)(4) + (0)(-) \\
&= 4
\end{aligned}
$$

Calculating the expected utilities (4/4)

What is the agent's expected utility of wearing pads and choosing the short route?

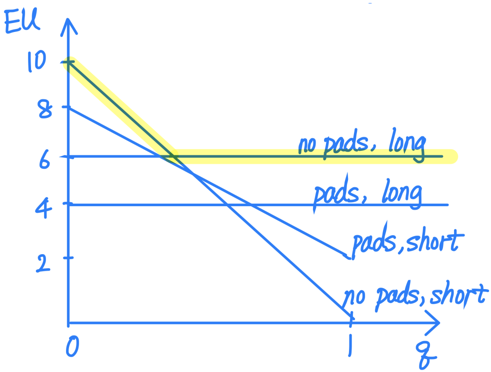


$\rightarrow$

$$
\begin{aligned}
EU(P,S) =& P(w_6|P \land S) * U(w_6) + P(w_7|P \land S) * U(w_7) \\
&= P(P \land S \land \neg A|P \land S) * U(w_6) \\
&\quad + P(P \land S \land A|P \land S) * U(w_7) \\
&= P(\neg A|P \land S) * U(w_6) P(A|P \land S) * U(w_7) \\
&= P(\neg A|S) * U(w_6) + P(A|S) * U(w_7) \\
&= (1-q)(8) + (q)(2) \\
&= 8 - 6q
\end{aligned}
$$

What should the robot do?

- Should it wear pads or not?
- Should it choose the short or the long route?



Optimal policy:

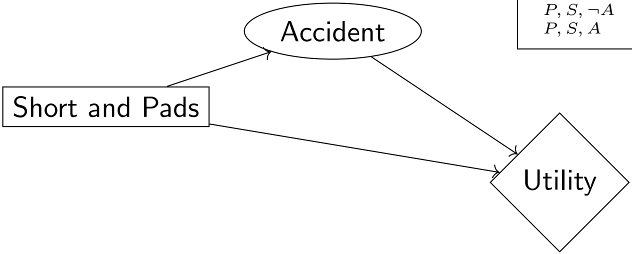
- If $q \leq 0.4$, no pads, short route
- If $q > 0.4$, no pads, long route

Robot decision network

- Since we can make the *Shorts* and *Pads* decision at the same time, we can combine the decision nodes into a single node.
- Domain of the combined node is the cross product of the domains of the original nodes

$$P(A|\neg S) = 0$$
$$P(A|S) = q$$

	$U(w_i)$
$\neg P, \neg S, \neg A$	6
$\neg P, \neg S, A$	-
$\neg P, S, \neg A$	10
$\neg P, S, A$	0
$P, \neg S, \neg A$	4
$P, \neg S, A$	-
$P, S, \neg A$	8
P, S, A	2

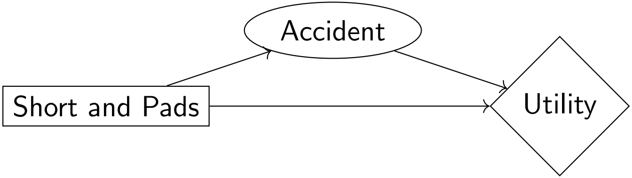


Variable Elimination for a Single-Stage Decision Network

In summary:

1. Prune all the nodes that are not ancestors of the utility node.
2. Sum out all chance nodes.
3. For the single remaining factor, return the maximum value and the assignment that gives the maximum value.

Define the Factors



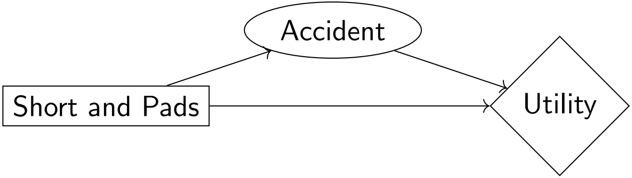
$f_1(A, S \wedge P)$:

A	$S \wedge P$	val
t	$S \wedge P$	q
f	$S \wedge P$	$1-q$
t	$S \wedge \neg P$	q
f	$S \wedge \neg P$	$1-q$
t	$\neg S \wedge P$	0
f	$\neg S \wedge P$	1
t	$\neg S \wedge \neg P$	0
f	$\neg S \wedge \neg P$	1

$u(A, S \wedge P)$:

A	$S \wedge P$	val
t	$S \wedge P$	2
f	$S \wedge P$	8
t	$S \wedge \neg P$	0
f	$S \wedge \neg P$	10
t	$\neg S \wedge P$	$-$
f	$\neg S \wedge P$	4
t	$\neg S \wedge \neg P$	$-$
f	$\neg S \wedge \neg P$	6

Sum out all chance nodes



Multiply the two factors.

$f_2(A, S \wedge P)$:

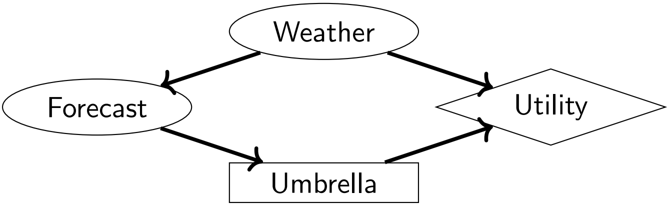
A	$S \wedge P$	val
t	$S \wedge P$	$2q$
f	$S \wedge P$	$8 - 8q$
t	$S \wedge \neg P$	0
f	$S \wedge \neg P$	$10 - 10q$
t	$\neg S \wedge P$	0
f	$\neg S \wedge P$	4
t	$\neg S \wedge \neg P$	0
f	$\neg S \wedge \neg P$	6

Sum out A from f_2.

$f_3(S \wedge P)$:

$S \wedge P$	val
$S \wedge P$	$8 - 6q$
$S \wedge \neg P$	$10 - 10q$
$\neg S \wedge P$	4
$\neg S \wedge \neg P$	6

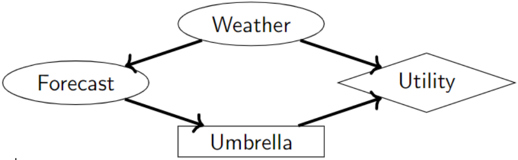
The Weather Decision Network



→ A chance node is a parent of a decision node.

We need to decide on whether to take or leave the umbrella based on the forecast (sunny, cloudy, rainy).

The conditional probabilities



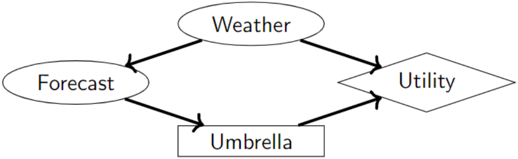
The Weather node:

Weather	P(Weather)
norain	0.7
rain	0.3

The Forecast node:

Weather	Forecast	P(Forecast ∣ Weather)
no_rain	sunny	0.7
no_rain	cloudy	0.2
no_rain	rainy	0.1
rain	sunny	0.15
rain	cloudy	0.25
rain	rainy	0.6

The utility function



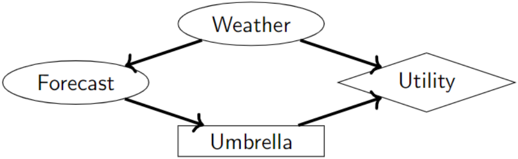
The Utility node:

Weather	Umbrella	u(Weather, Umbrella)
no_rain	take_it	20
no_rain	leave_it	100
rain	take_it	70
rain	leave_it	0

Policies

- A **policy** specifies what the agent should do under all contingencies.
- For each decision variable, a policy specifies a value for the decision variable for each assignment of values to its parents.

Question: Number of Policies



Q #3: For the weather decision network, how many possible policies are there?

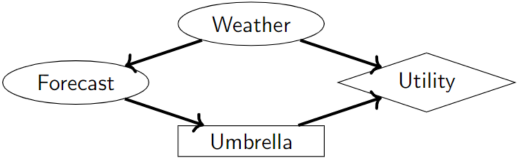
(A) 1

(B) 3

(C) 6

(D) 8

Question: Number of Policies



Q #3: For the weather decision network, how many possible policies are there?

(A) 1

(B) 3

(C) 6

(D) 8

→ (D) is correct. There are $2^3 = 8$ possible policies. Forecast has 3 values and there are 2 possible decisions.

Solving the weather problem

Two approaches:

- Compute the expected utility of each policy, and choose the policy that maximizes the expected utility.
- Use the variable elimination algorithm.

Question: The Expected Utility of a Policy

Q #4: Consider the policy π_1 below.

- take the umbrella if the forecast is **cloudy**, and
- leave the umbrella at home otherwise.

What is the expected utility of the policy π_1?

W	U	u(W, U)
no_rain	take_it	20
no_rain	leave_it	100
rain	take_it	70
rain	leave_it	0

W	F	P(F | W)
no_rain	sunny	0.7
no_rain	cloudy	0.2
no_rain	rainy	0.1
rain	sunny	0.15
rain	cloudy	0.25
rain	rainy	0.6

(A) -40.21
(B) 32.06
(C) 64.05
(D) 72.62

W	P(W)
norain	0.7
rain	0.3

Question: The Expected Utility of a Policy

Q #4: Consider the policy π_1 below.

- take the umbrella if the forecast is **cloudy**, and
- leave the umbrella at home otherwise.

What is the expected utility of the policy π_1?

W	U	u(W, U)
no_rain	take_it	20
no_rain	leave_it	100
rain	take_it	70
rain	leave_it	0

W	F	P(F | W)
no_rain	sunny	0.7
no_rain	cloudy	0.2
no_rain	rainy	0.1
rain	sunny	0.15
rain	cloudy	0.25
rain	rainy	0.6

(A) -40.21
(B) 32.06
(C) 64.05
(D) 72.62

W	P(W)
norain	0.7
rain	0.3

→ (C). $EU(\pi_1) = \sum_{w,f} P(w)P(f|w)u(w, \pi_1(f)) = 64.05$

Question: The Expected Utility of a Policy

Q #5: Consider the policy π_2 below:

- take the umbrella if the forecast is **rainy**, and
- leave the umbrella at home otherwise.

What is the expected utility of the policy π_2?

W	U	u(W, U)
no_rain	take_it	20
no_rain	leave_it	100
rain	take_it	70
rain	leave_it	0

W	F	P(F | W)
no_rain	sunny	0.7
no_rain	cloudy	0.2
no_rain	rainy	0.1
rain	sunny	0.15
rain	cloudy	0.25
rain	rainy	0.6

(A) 62.5
(B) 77
(C) 83
(D) 90.2

W	P(W)
norain	0.7
rain	0.3

Question: The Expected Utility of a Policy

Q #5: Consider the policy π_2 below:

- take the umbrella if the forecast is **rainy**, and
- leave the umbrella at home otherwise.

What is the expected utility of the policy π_2?

W	U	u(W, U)
no_rain	take_it	20
no_rain	leave_it	100
rain	take_it	70
rain	leave_it	0

W	F	P(F | W)
no_rain	sunny	0.7
no_rain	cloudy	0.2
no_rain	rainy	0.1
rain	sunny	0.15
rain	cloudy	0.25
rain	rainy	0.6

W	P(W)
norain	0.7
rain	0.3

(A) 62.5
(B) 77
(C) 83
(D) 90.2

$\rightarrow$ (B). $EU(\pi_2) = \sum_{w,f} P(w)P(f|w)u(w, \pi_2(f)) = 77$

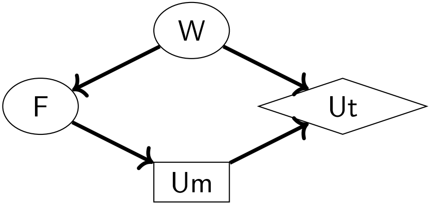
Variable elimination algorithm

1. Remove all variables that are not ancestors of the utility node.
2. Define a factor for every non-decision node.
3. While there are decision nodes remaining
   3.1 Eliminate each random variable that is not a parent of a decision node.
   3.2 Find the optimal policy for the last decision and eliminate the decision variable.
4. Return the optimal policies.
5. Determine agent's expected utility following the optimal policy by eliminating all the remaining random variables.

Applying VEA: Step 1

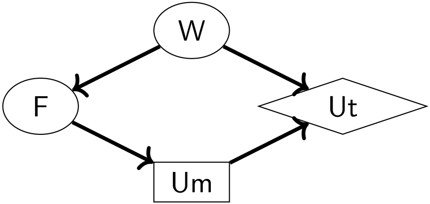
Step 1: Remove all variables that are not ancestors of the utility node.

Every variable is an ancestor of the utility node.
There's nothing to be done.



Applying VEA: Step 2

Step 2: Define three factors f_1 for Weather, f_2 for Forecast, and f_3 for the Utility.



W	value
norain	0.7
rain	0.3

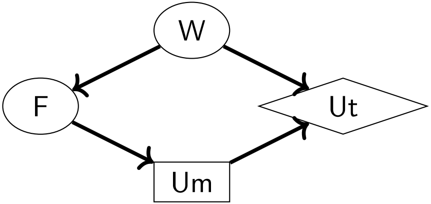
Table: Factor $f_1(W)$

W	F	value
norain	sunny	0.7
norain	cloudy	0.2
norain	rainy	0.1
rain	sunny	0.15
rain	cloudy	0.25
rain	rainy	0.6

Table: Factor $f_2(W, F)$

Applying VEA: Step 2

Step 2: Define three factors f_1 for Weather, f_2 for Forecast, and f_3 for the Utility (continued).



W	U	value
norain	takeit	20
norain	leaveit	100
rain	takeit	70
rain	leaveit	0

Table: Factor $f_3(W, U)$

Applying VEA: Step 3.1

Step 3.1: Weather is not a parent of any decision node.

Eliminate Weather.

Multiply all the factors that contain Weather.

$$f_1(W) \times f_2(W,F) \times f_3(W,U) = f_4(W,F,U)$$

Sum out Weather from $f_4(W,F,U)$.

$$\sum_W f_4(W,F,U) = f_5(F,U)$$

Applying VEA: Step 3.1 Factor f_4

W	F	U	value
norain	sunny	takeit	9.8
norain	sunny	leaveit	49
norain	cloudy	takeit	2.8
norain	cloudy	leaveit	14
norain	rainy	takeit	1.4
norain	rainy	leaveit	7
rain	sunny	takeit	3.15
rain	sunny	leaveit	0
rain	cloudy	takeit	5.25
rain	cloudy	leaveit	0
rain	rainy	takeit	12.6
rain	rainy	leaveit	0

Table: Factor $f_4(W, F, U)$

Applying VEA: Step 3.1 Factor f_5

F	U	value
sunny	takeit	12.95
sunny	leaveit	49
cloudy	takeit	8.05
cloudy	leaveit	14
rainy	takeit	14
rainy	leaveit	7

Table: Factor $f_5(F, U)$

Applying VEA: Step 3.2

Step 3.2: Find the optimal policy for Umbrella.

F	U	value
sunny	leaveit	49
cloudy	leaveit	14
rainy	takeit	14

Table: Finding the optimal policy for Umbrella from $f_5(F, U)$

F	U
sunny	leaveit
cloudy	leaveit
rainy	takeit

Table: Optimal policy for Umbrella

F	value
sunny	49
cloudy	14
rainy	14

Table: Factor $f_6(F)$ w/o Umbrella

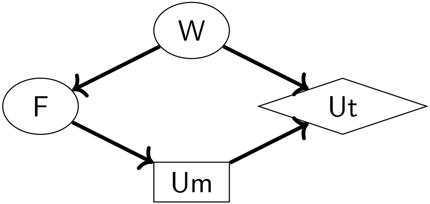
Agent's Expected Utility of the Optimal Policy

Sum out Forecast from $f_6(F)$ to produce $f_7()$.

value
77

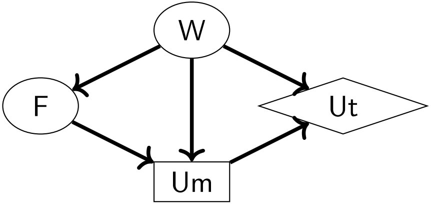
Table: Factor $f_7()$

Following the optimal policy, the agent's expected utility is 77.



The Value of Information

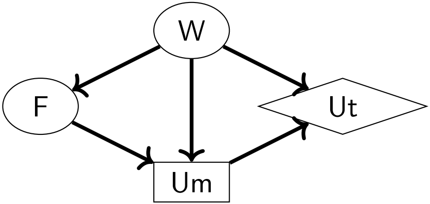
What if I can observe the Weather directly?



Applying VEA: Step 1

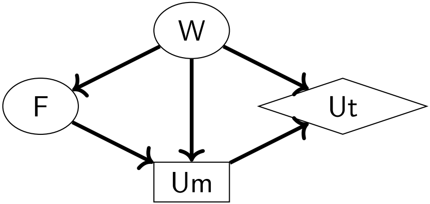
Step 1: Remove all variables that are not ancestors of the utility node.

Every variable is an ancestor of the utility node.
There's nothing to be done.



Applying VEA: Step 2

Step 2: Define three factors. $f_1(W)$ for Weather, $f_2(W, F)$, and $f_3(W, U)$ for the Utility.



W	value
norain	0.7
rain	0.3

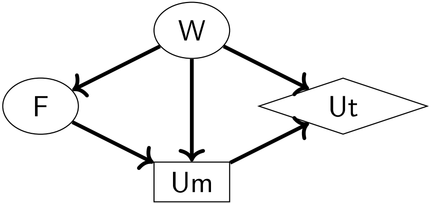
Table: Factor $f_1(W)$

W	F	value
norain	sunny	0.7
norain	cloudy	0.2
norain	rainy	0.1
rain	sunny	0.15
rain	cloudy	0.25
rain	rainy	0.6

Table: Factor $f_2(W, F)$

Applying VEA: Step 2 (continued)

Step 2: Define three factors. $f_1(W)$ for Weather, $f_2(W, F)$, and $f_3(W, U)$ for the Utility.



W	U	value
norain	takeit	20
norain	leaveit	100
rain	takeit	70
rain	leaveit	0

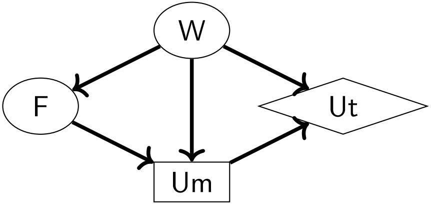
Table: Factor $f_3(W, U)$

Applying VEA: Step 3.1

Step 3.1: Eliminate any variable that is not a parent of a decision node.

Weather and Forecast are both parents of Umbrella.

Nothing needs to be done for this step.



Applying VEA: Step 3.2

Step 3.2: Determine the optimal policy for Umbrella.

Find a factor that contains the decision node (Umbrella). All the other variables in the factor must be the decision node's parents.

$f_3(W, U)$ satisfies the requirements.

W	U	value
norain	takeit	20
norain	leaveit	100
rain	takeit	70
rain	leaveit	0

Table: Factor $f_3(W, U)$

Applying VEA: Steps 3.2

Step 3.2: Determine the optimal policy for Umbrella.

W	U	value
norain	leaveit	100
rain	takeit	70

Table: Finding the optimal policy for Umbrella

W	U
norain	leaveit
rain	takeit

Table: The optimal policy for Umbrella

W	value
norain	100
rain	70

Table: Factor $f_4(W)$ without Umbrella

Agent's Expected Utility Following the Optimal Policy

Determine agent's expected utility if they follow the optimal policy.

Eliminate Forecast.

$$\sum_{F} f_2(F, W) = f_5(W).$$

W	value
norain	1
rain	1

Table: Factor $f_5(W)$

Agent's Expected Utility Following the Optimal Policy

Determine agent's expected utility if they follow the optimal policy.

Eliminate Weather.

Multiply all the factors containing Weather.

$$f_1(W) \times f_4(W) \times f_5(W) = f_6(W).$$

Sum out Weather from $f_6(W)$ to produce $f_7()$.

$$\sum_W f_6(W) = f_7().$$

W	value
norain	70
rain	21

Table: Factor $f_6(W)$

value
91

Table: Factor $f_7()$

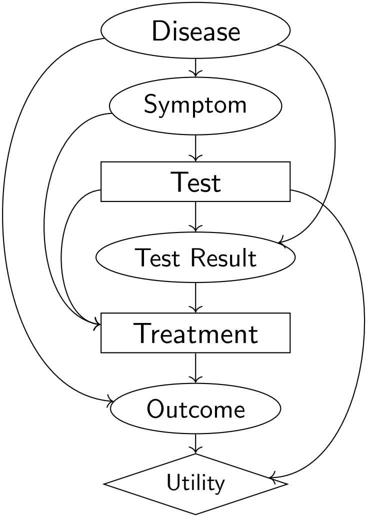
A Medical Diagnosis Scenario

Consider a simple medical diagnosis scenario. A doctor needs to make two decisions: choosing a test to perform and decides on a treatment based on the test results. The reason for performing a test is to obtain information (the test results) that may be useful for determining the treatment. It is often a good idea to test, even if testing itself may harm the patient.

The doctor can decide on which test to perform based on the patient's symptom. When deciding the treatment, the information available will be the patient's symptom, the tests performed, and the test results. The test results depend on the disease and what test was performed. The treatment outcome depends on the disease and the treatment performed. The patient's utility includes costs of tests and treatments, the pain and inconvenience to the patient in the short term, and the long-term prognosis.

(Example 9.12 in Poole and Mackworth)

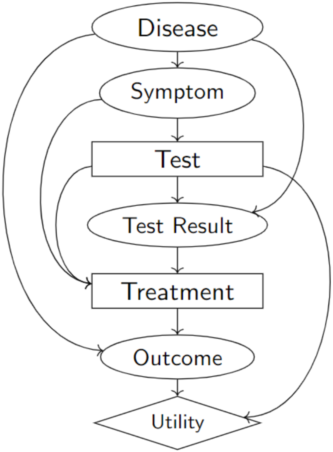
Decision Network for the Diagnosis Scenario



VEA: Step 1

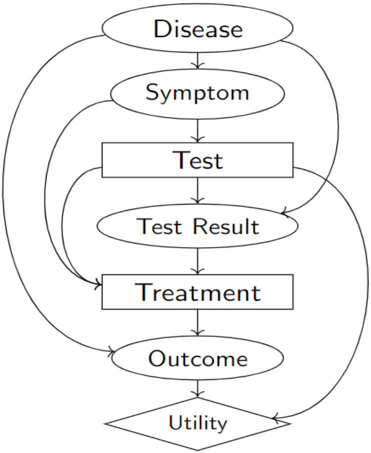
Step 1: Remove all variables that are not ancestors of the utility node.

All the variables are ancestors of the utility node. There is nothing to be done for this step.



VEA: Step 2

Step 2: Define one factor for every random variable (chance node).



D	value
1	0.79
0	0.21

Table: Disease: $f_1(D)$

D	S	value
1	1	0.89
1	0	0.11
0	1	0.27
0	0	0.73

Table: Symptom: $f_2(D, S)$

VEA: Step 2

Define one factor for every random variable (chance node).

D	T	TR	value
1	1	1	0.22
1	1	0	0.78
1	0	1	0.55
1	0	0	0.45
0	1	1	0.91
0	1	0	0.09
0	0	1	0.53
0	0	0	0.47

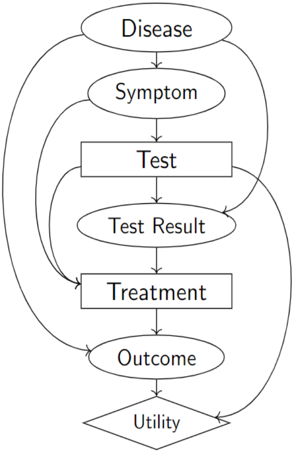
Table: Test Result: $f_3(D, T, TR)$

D	Tr	O	value
1	1	1	0.59
1	1	0	0.41
1	0	1	0.16
1	0	0	0.84
0	1	1	0.93
0	1	0	0.07
0	0	1	0.65
0	0	0	0.35

Table: Outcome: $f_4(D, Tr, O)$

VEA: Step 2

Define one factor for the utility node.



T	O	value
1	1	900
1	0	600
0	1	1000
0	0	700

Table: Utility: $f_5(T, O)$

VEA: Step 3

There are still decision nodes remaining.
Eliminate every random variable that is not a parent of a decision node.

There are two such random variables: Disease and Outcome.

Let's eliminate Outcome first.

VEA: Step 3.1

Step 3.1: Eliminate Outcome.

Multiply all the factors that contain Outcome.

$$f_4(D, Tr, O) \times f_5(T, O)$$
$$= f_6(D, O, T, Tr)$$

Sum out Outcome.

$$\sum_{O} f_6(D, O, T, Tr) = f_7(D, T, Tr)$$

The new list of factors:
$f_1(D), f_2(D, S), f_3(D, T, TR), f_7(D, T, Tr)$.

VEA: Step 3.1 Factors

D	O	T	Tr	value
1	1	1	1	531
1	1	1	0	144
1	1	0	1	590
1	1	0	0	160
1	0	1	1	246
1	0	1	0	504
1	0	0	1	287
1	0	0	0	588
0	1	1	1	837
0	1	1	0	585
0	1	0	1	930
0	1	0	0	650
0	0	1	1	42
0	0	1	0	210
0	0	0	1	49
0	0	0	0	245

Table: Factor $f_6(D, O, T, Tr)$

D	T	Tr	value
1	1	1	777
1	1	0	648
1	0	1	877
1	0	0	748
0	1	1	879
0	1	0	795
0	0	1	979
0	0	0	895

Table: Factor $f_7(D, T, Tr)$

VEA: Step 3.1

Step 3.1: Eliminate Disease.

Multiply all the factors that contain Disease.

$$f_1(D) \times f_2(D,S) \times f_3(D,T,TR) \times f_7(D,T,Tr)$$
$$= f_8(D,S,T,TR,Tr)$$

Sum out Disease.

$$\sum_D f_8(D,S,T,TR,Tr) = f_9(S,T,TR,Tr)$$

The new list of factors: $f_9(S,T,TR,Tr)$.

VEA: Step 3.1 Factors

D	S	T	TR	Tr	value
1	1	1	1	1	120.2
1	1	1	1	0	100.2
1	1	1	0	1	426.1
1	1	1	0	0	355.4
1	1	0	1	1	339.1
1	1	0	1	0	289.3
1	1	0	0	1	277.5
1	1	0	0	0	236.7
1	0	1	1	1	14.85
1	0	1	1	0	12.39
1	0	1	0	1	52.67
1	0	1	0	0	43.92
1	0	0	1	1	41.92
1	0	0	1	0	35.75
1	0	0	0	1	34.30
1	0	0	0	0	29.25

Table: Factor $f_8(D, S, T, TR, Tr)$ first half

D	S	T	TR	Tr	value
0	1	1	1	1	45.35
0	1	1	1	0	41.02
0	1	1	0	1	4.486
0	1	1	0	0	4.057
0	1	0	1	1	29.42
0	1	0	1	0	26.90
0	1	0	0	1	26.09
0	1	0	0	0	23.85
0	0	1	1	1	122.6
0	0	1	1	0	110.9
0	0	1	0	1	12.13
0	0	1	0	0	10.97
0	0	0	1	1	79.54
0	0	0	1	0	72.72
0	0	0	0	1	70.54
0	0	0	0	0	64.49

Table: Factor $f_8(D, S, T, TR, Tr)$ second half

VEA: Step 3.1 Factors

S	T	TR	Tr	value
1	1	1	1	165.5
1	1	1	0	141.3
1	1	0	1	430.6
1	1	0	0	359.4
1	0	1	1	368.6
1	0	1	0	316.2
1	0	0	1	303.6
1	0	0	0	260.5
0	1	1	1	137.5
0	1	1	0	123.3
0	1	0	1	64.79
0	1	0	0	54.89
0	0	1	1	121.5
0	0	1	0	108.5
0	0	0	1	104.8
0	0	0	0	93.74

Table: Factor $f_9(S, T, TR, Tr)$

VEA: Step 3.2 Maximize Utility

S	T	TR	Tr	value
1	1	1	1	165.5
1	1	1	0	141.3
1	1	0	1	430.6
1	1	0	0	359.4
1	0	1	1	368.6
1	0	1	0	316.2
1	0	0	1	303.6
1	0	0	0	260.5
0	1	1	1	137.5
0	1	1	0	123.3
0	1	0	1	64.79
0	1	0	0	54.89
0	0	1	1	121.5
0	0	1	0	108.5
0	0	0	1	104.8
0	0	0	0	93.74

Table: Factor $f_9(S, T, TR, Tr)$

VEA: Step 3.2

Step 3.2: Find the optimal policy for Treatment and eliminate it.

S	T	TR	Tr
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	1
0	1	1	1
0	1	0	1
0	0	1	1
0	0	0	1

Table: The optimal policy for Treatment

S	T	TR	value
1	1	1	165.5
1	1	0	430.6
1	0	1	368.6
1	0	0	303.6
0	1	1	137.5
0	1	0	64.79
0	0	1	121.5
0	0	0	104.8

Table: Factor $f_{10}(S, T, TR)$

The new list of factors: $f_{10}(S, T, TR)$

VEA: Step 3.1

There are still decision nodes remaining.
Eliminate every random variable that is not a parent of a decision node.

There is one such random variable: Test Result.

Let's eliminate Test Result.

VEA: Step 3.1

Step 3.1: Eliminate Test Result.

Multiply all the factors that contain Test Result.
Sum out Test Result.

$$\sum_{TR} f_{10}(S, T, TR) = f_{11}(S, T)$$

S	T	value
1	1	596.1
1	0	672.2
0	1	202.3
0	0	226.3

Table: Factor $f_{11}(S, T)$

The new list of factors: $f_{11}(S, T)$.

VEA: Step 3.2

Step 3.2: Find the optimal policy for Test and eliminate it. Take the maximum utility for T given S.

S	T	value
1	1	596.1
1	0	672.2
0	1	202.3
0	0	226.3

Table: Factor $f_{11}(S, T)$

VEA: Step 3.2

Step 3.2: Find the optimal policy for Test and eliminate it.

S	T	value
1	0	672.2
0	0	226.3

Table: Finding the optimal policy for Test

S	T
1	0
0	0

Table: The optimal policy for Test

S	value
1	672.2
0	226.3

Table: Factor $f_{12}(S)$ without Test

The new list of factors: $f_{12}(S)$

Agent's Expected Utility following the Optimal Policy

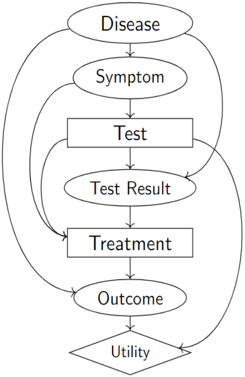
Sum out all the remaining random variables.

Sum out Symptom.

$$\sum_S f_{12}(S) = f_{13}()$$

value
898.5

Table: Factor $f_{13}()$



Regardless of the Symptom, do not take any test.
Go straight to treatment no matter what.

Following the optimal policy, the agent's expected utility is 898.5.

Revisit Learning Goals

- Model a one-off decision problem by constructing a decision network containing nodes, arcs, conditional probability distributions, and a utility function.
- Determine the optimal policy of a decision network by computing the expected utility of every policy.
- Determine the optimal policy of a decision network by applying the variable elimination algorithm.
- Given a decision network with a single decision, determine the optimal policy and the expected utility of the optimal policy by enumerating all the policies.
- Given a decision network with a single decision, determine the optimal policy and the expected utility of the optimal policy by applying the variable elimination algorithm.