CS 486/686
Uninformed Search

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Lecture 2

RN 3.1–3.4

Learning goals

- Formulate problems as search
- Trace DFS, BFS, IDS
- Analyze: space, time, completeness, optimality
- Pick the right algorithm

Heads-up: Chat 1 is live

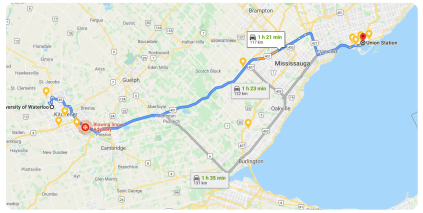
Sign up: andromeda-208.cs.uwaterloo.ca

- Use your **WatIAM @uwaterloo.ca** email (not your friendly one)
- Verify the email Chrysalis sends; you'll be auto-added to CS 486/686
- **Chat 1** covers L1 + L2 — due **Tue May 26, 11:59 PM** *(one-time extension for late enrollees)*
- Issues? Private Piazza post to Prashanth Arun & Robert Cai (Chrysalis designers)

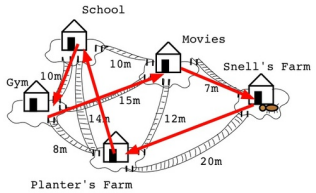
Full instructions: course page → Chat Assignments with Chrysalis.

Where does search show up?

Robotics



Route planning

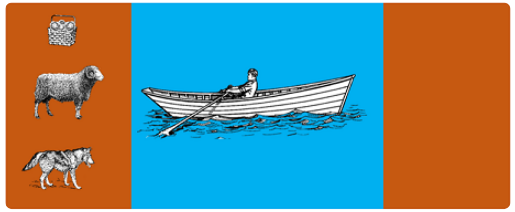


TSP

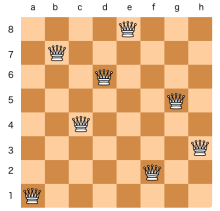
$$((a \wedge b) \vee c) \wedge d \vee (\neg e)$$

SAT

8-puzzle



River crossing



N-queens

• • •

And many more

TSP: Quanta. SAT / FCC auction: UBC news.

Why search?

- Sequential decisions
- No closed-form algorithm
- Easy to verify, hard to find
- Systematic, not guesswork

What is a search problem?

Five components:

- **States**
- **Initial state**
- **Goal test**
- **Successor function**
- **Cost** (optional)

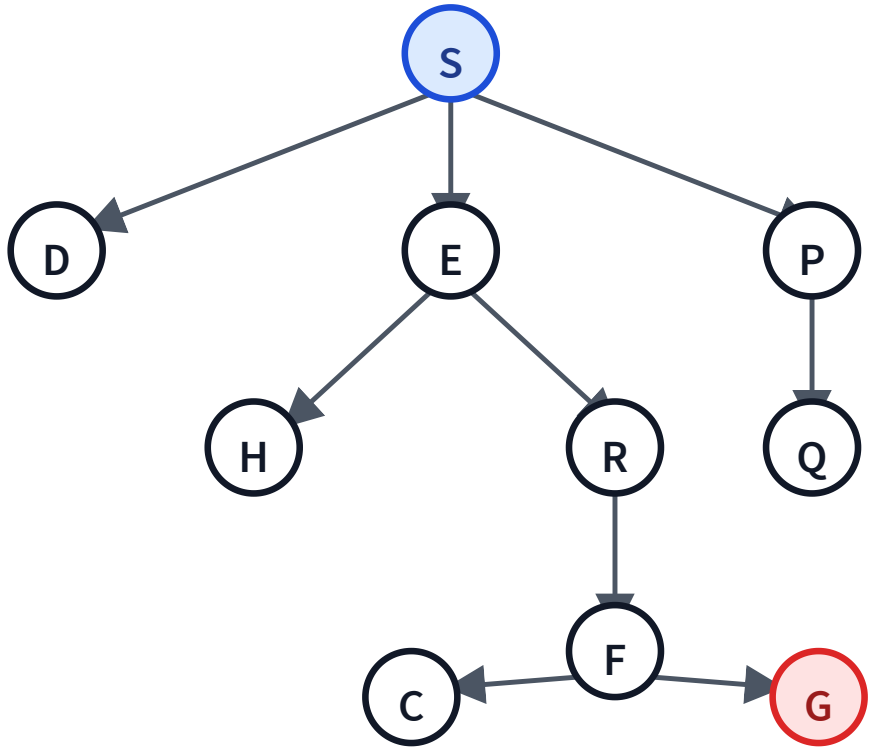
Solution = path from start to goal.

State spaces are huge

1.8×10^5	10^{13}	4.3×10^{19}
8-puzzle	**15-puzzle**	**Rubik's cube**
3×3 sliding tile	4×4 sliding tile	3×3×3

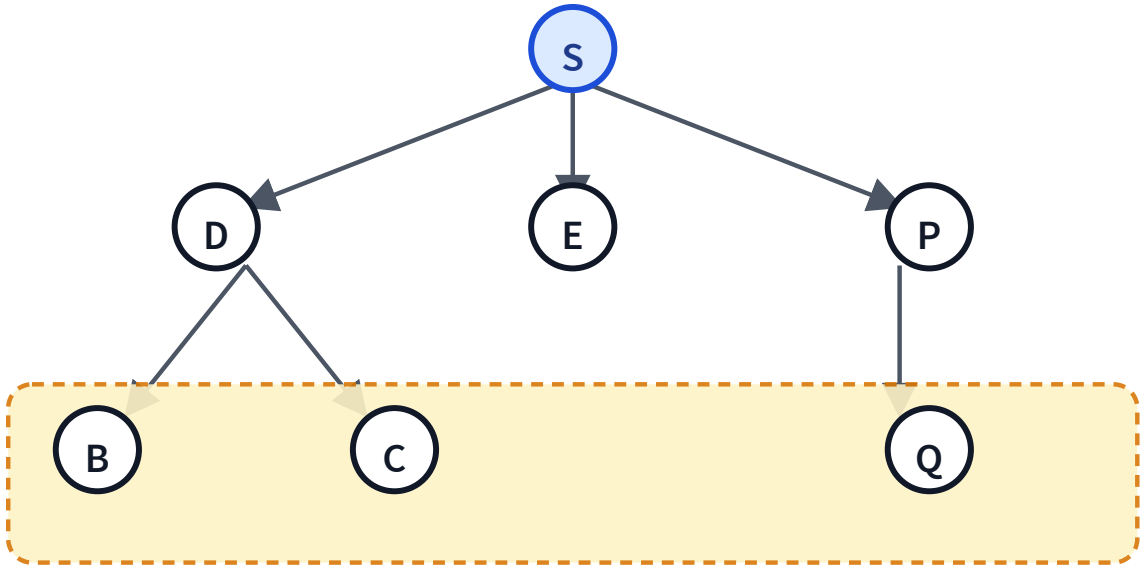
Too many states to enumerate. We need **clever search**.

Search graph



- States → **nodes**
- Actions → **edges**
- Goal: find a path **S → G**

Search tree & frontier



- Don't enumerate the graph
- Grow a tree from S, one path at a time
- **Frontier** = paths to expand next

Example: 8-puzzle

Initial

5	3	
8	7	6
2	4	1

Goal

1	2	3
4	5	6
7	8	

- **State:** 9 positions, `0` = empty
- **Initial:** `530,876,241`
- **Goal:** `123,456,780`
- **Successor:** slide adjacent tile into empty
- **Cost:** 1 per move

181,440 reachable states — too many to enumerate.

Quick check: successors

Q. Which is a successor of `530,876,241`?

A. `350,876,241`

B. `536,870,241`

C. `537,806,241`

D. `538,076,241`

Initial

5	3	
8	7	6
2	4	1

B. Slide the 6 up.

Multiple formulations possible

- State def → nodes
- Successor → edges
- Minimize both

Same 8-puzzle, different state

By grid

5	3	
8	7	6
2	4	1

9 cells, 0–8

VS

By coordinates

$$(x_0, y_0), (x_1, y_1), (x_2, y_2), \ldots, (x_8, y_8)$$

9 (x,y) pairs — ~2× the space

Successor swaps empty tile's coords with a neighbour's.

Generic search algorithm

```
// Inputs: graph, start node s, goal test goal(n)
function Search(graph, s, goal):
    frontier ← { <s> }
    while frontier is not empty:
        select and remove a path <n_0, …, n_k> from frontier
        if goal(n_k):
            return <n_0, …, n_k>
        for each neighbour n of n_k:
            add <n_0, …, n_k, n> to frontier
    return “no solution”
```

expand: **pop path** → get neighbours → push extended paths

One template, three algorithms

DFS, BFS, IDS all use this skeleton.

They differ only in **which path** they pull from the frontier.

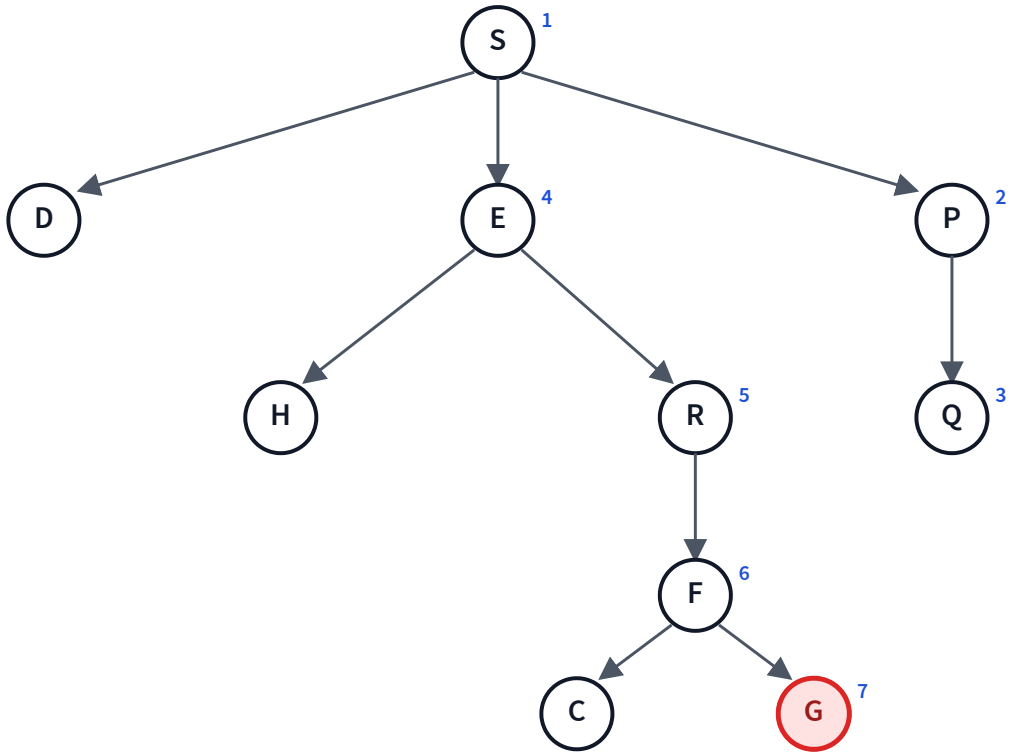
Depth-first search (DFS)

- Frontier = **stack** (LIFO)
- Pop newest
- Dive deep, then backtrack

Trace: DFS

Push children alphabetically. Step numbers = order of expansion.

```
Frontier:
```

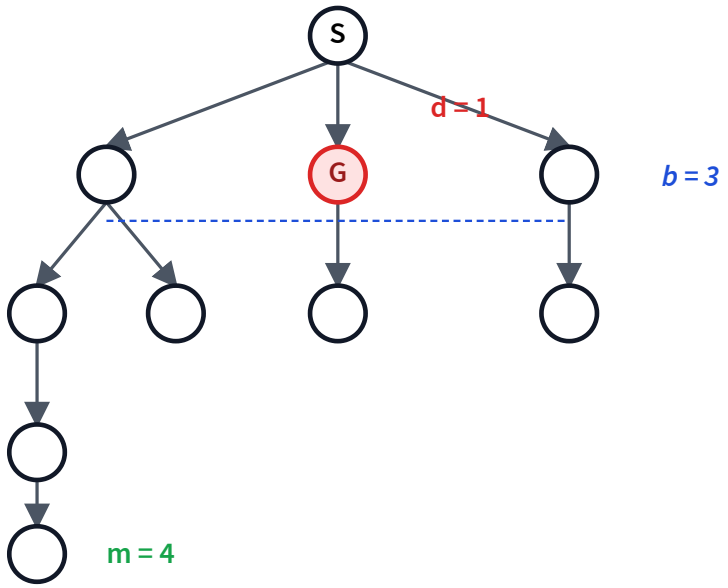


Evaluating search algorithms

Four questions we'll ask about every algorithm:

- **Space** — how much memory? (worst-case frontier size)
- **Time** — how many nodes does it visit? (worst case)
- **Complete?** — is it guaranteed to find a goal if one exists?
- **Optimal?** — does it return the cheapest (or shallowest) goal?

Tree parameters: b, m, d



b — branching factor

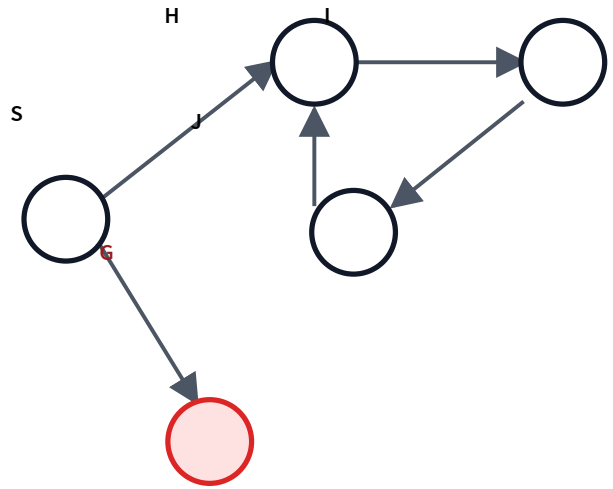
d — depth of shallowest goal

m — max depth of tree

DFS — properties

Space	$O(bm)$ *linear*
Time	$O(b^m)$ *exponential*
Complete?	**No**
Optimal?	**No**

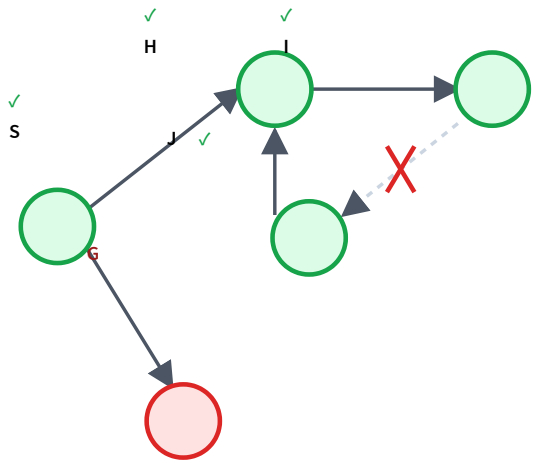
DFS fails on cycles



$H \to I \to J \to H$ traps DFS forever.

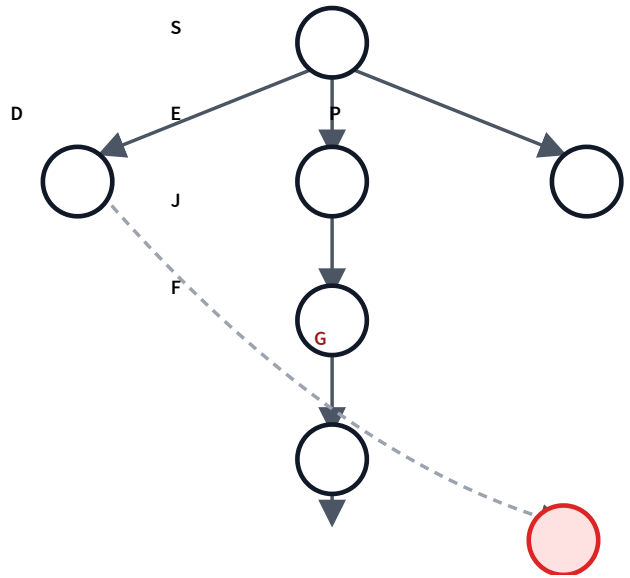
Fix: cycle pruning

Keep a **visited** set. Skip states we've already expanded.



Order: S → H → I → J → *back to H? skip!* → G.

DFS takes the first goal, not the best



Returns the long path; ignores the shorter $S \rightarrow D \dashrightarrow G$.

When to use DFS

Good fit

- Memory limited
- Many goals exist
- Deep solutions

Bad fit

- Cycles
- Shallow solutions
- Need optimality

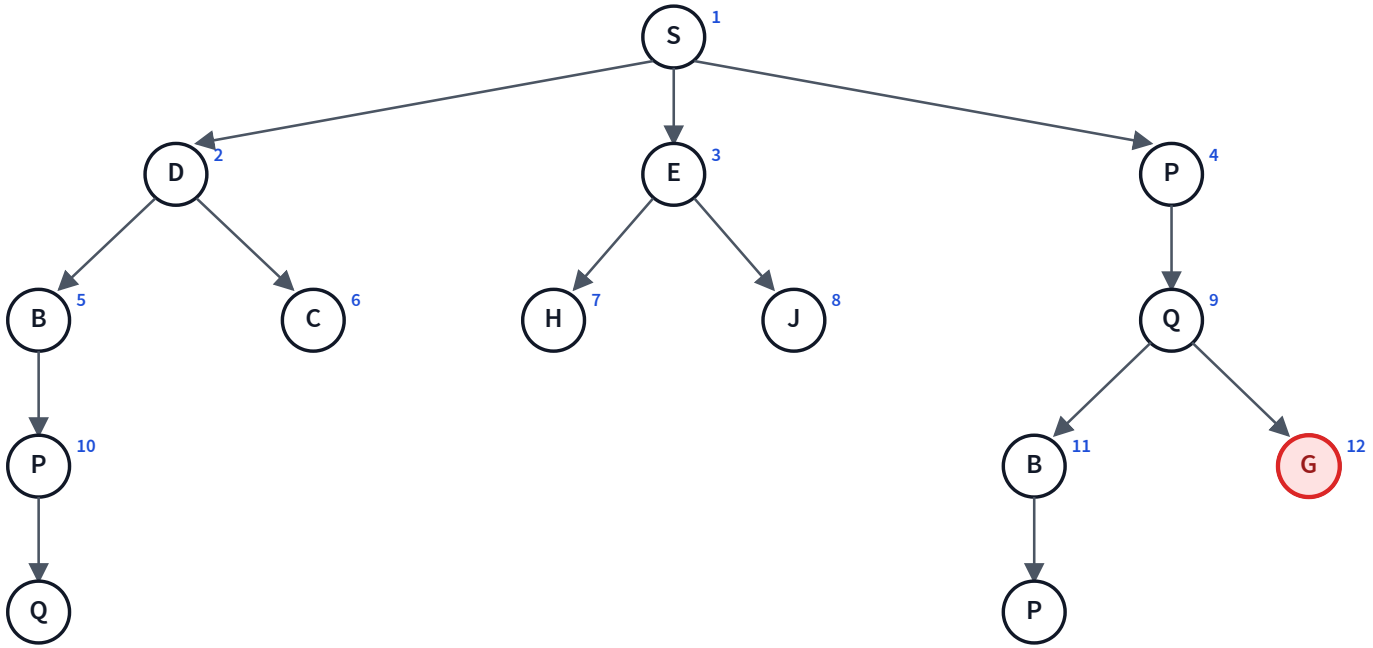
Breadth-first search (BFS)

- Frontier = **queue** (FIFO)
- Pop oldest
- Level by level

Trace: BFS

Pop the oldest path. Step number = order of expansion. Multiple paths to *P* and *B*.

```
Frontier:
```



BFS — properties

Space	$O(b^d)$ *exponential*
Time	$O(b^d)$ *exponential*
Complete?	**Yes**
Optimal?	Shallowest goal *(optimal at equal cost)*

When to use BFS

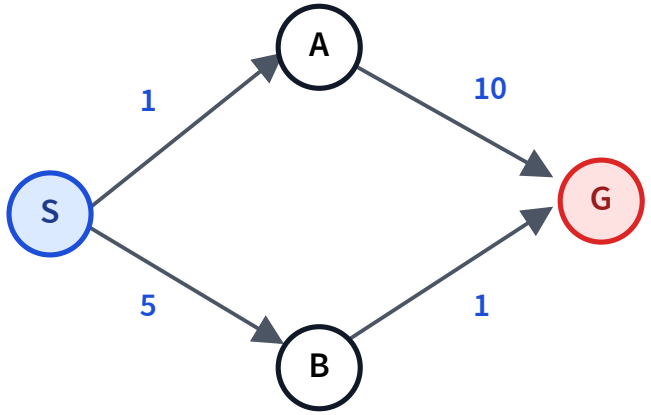
Good fit

- Memory plentiful
- Want fewest arcs
- Cycles possible

Bad fit

- Deep solutions
- Large branching
- Graph generated on the fly

What if edges have different costs?



S→A→G: cost 11. S→B→G: cost 6.
BFS picks the wrong one (it's fewer hops, but pricier).

Uniform-cost search (UCS)

- Frontier = **priority queue**
- Key = path cost so far
- Pop the **cheapest** path first

Complete + optimal when all costs are positive.

When every edge has cost 1, UCS = BFS. (We'll use BFS in this lecture for that reason.)

BFS vs DFS — Q1

Q. Memory is very limited.

A. BFS

B. DFS

C. Both are fine

D. Neither

> **B — DFS.** DFS: $O(bm)$ vs BFS: $O(b^d)$.

BFS vs DFS — Q2

Q. All solutions are deep in the search tree.

A. BFS

B. DFS

C. Both are fine

D. Neither

> **B — DFS.** Dives down long paths instead of enumerating every shallower level.

BFS vs DFS — Q3

Q. The graph contains cycles.

A. BFS

B. DFS

C. Both are fine

D. Neither

> **A — BFS.** DFS can *fail to find* the goal (gets stuck going deeper into the cycle); BFS still finds any goal at finite depth. (BFS does waste work without cycle pruning — but it's complete.)

BFS vs DFS — Q4

Q. The branching factor is huge or infinite.

A. BFS

B. DFS

C. Both are fine

D. Neither

> **B — DFS.** BFS's frontier blows up with b; DFS stays linear.

BFS vs DFS — Q5

Q. You must find the *shallowest* goal.

A. BFS

B. DFS

C. Both are fine

D. Neither

> **A — BFS.** BFS guarantees the shallowest goal.

BFS vs DFS — Q6

Q. All solutions are very shallow.

A. BFS

B. DFS

C. Both are fine

D. Neither

> **A — BFS.** Checks shallow neighbours before diving deep.

BFS vs DFS tradeoff

	BFS	DFS
Space	$O(b^d)$	$O(bm)$
Complete?	Yes	No

Can we have **both**?

Why $O(b^d)$ matters

Branching factor $b = 10$, 1 state = 8 bytes, computer doing 10^9 states/sec.

Depth d	States b^d	BFS memory	Wall-clock time
4	10^4	*80 KB*	*10 µs*
8	10^8	*800 MB*	*0.1 s*
12	10^{12}	*8 TB*	*~17 min*
16	10^{16}	*80 PB*	*~4 months*
20	10^{20}	*800 EB*	*~3000 years*

Doubling d doesn't make the problem twice as hard. It makes it 10,000× harder.

Iterative-Deepening Search (IDS)

For $\ell = 1, 2, 3, \ldots$, run DFS up to depth ℓ.

Stop when a goal is found.

*Each iteration is **depth-limited search (DLS)**.*

Trace: IDS

Same graph as BFS. Each iteration is depth-limited DFS.

```
Depth limit:
```

IDS — properties

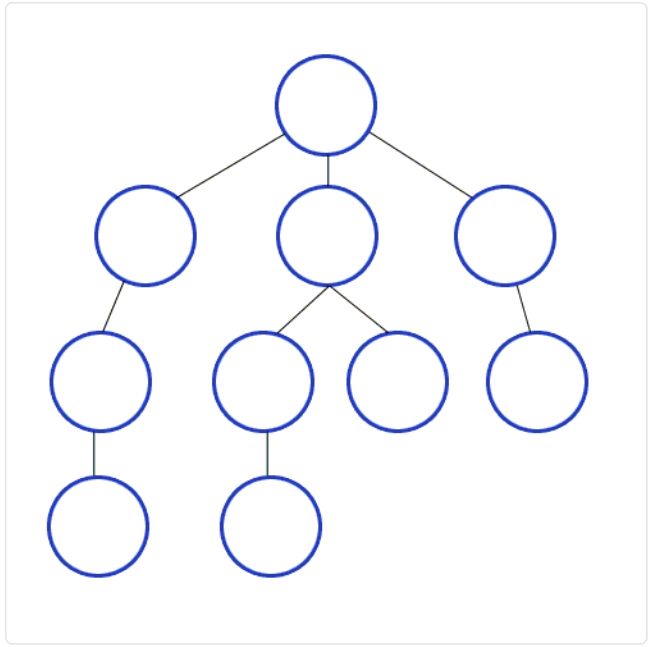
Space	$O(bd)$ *linear*
Time	$O(b^d)$ *exponential*
Complete?	**Yes**
Optimal?	Shallowest goal

Summary: DFS vs BFS vs IDS

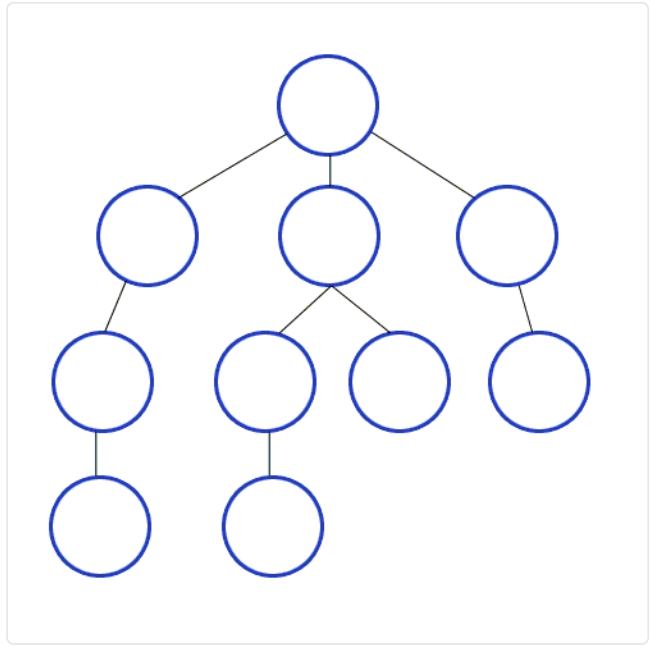
	DFS	BFS	IDS
Frontier	Stack (LIFO)	Queue (FIFO)	Stack, repeated at deeper limits
Space	$O(bm)$	$O(b^d)$	$O(bd)$
Time	$O(b^m)$	$O(b^d)$	$O(b^d)$
Complete?	**No**	**Yes**	**Yes**
Optimal?	**No**	Shallowest goal	Shallowest goal

b = branching factor; m = max depth of the search tree; d = depth of the shallowest goal.

DFS vs BFS: exploration order



DFS — dives deep first



BFS — expands by level

Animations by Mre, Wikimedia Commons (CC BY-SA 3.0).

Learning goals (recap)

✓ Formulate problems as search

✓ Trace DFS, BFS, IDS

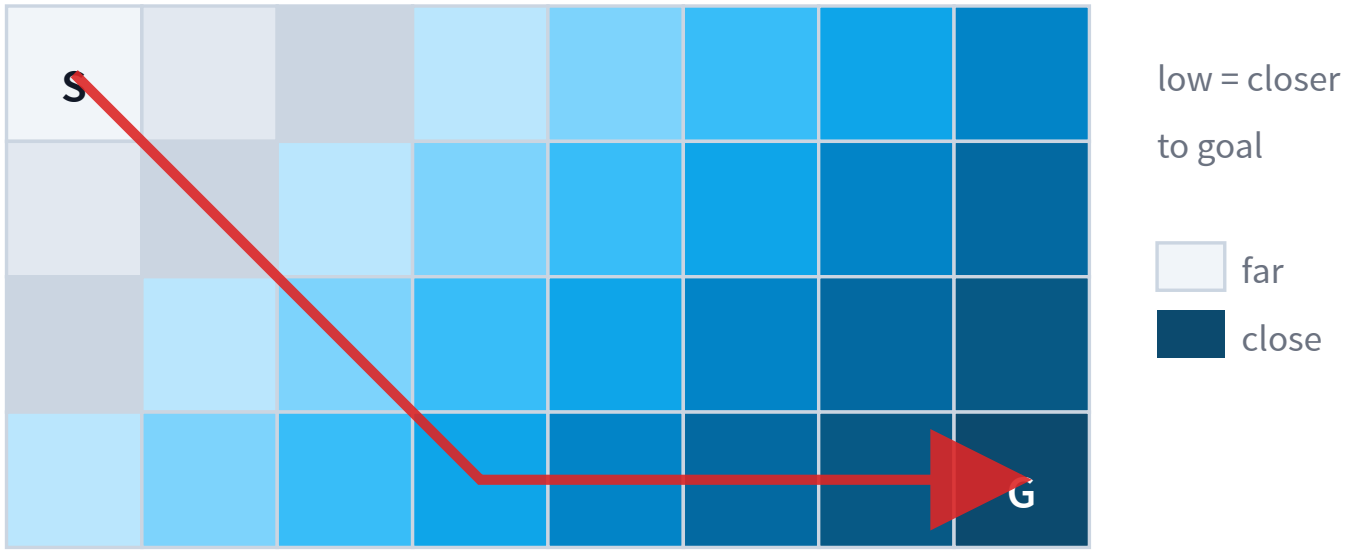
✓ Analyze: space, time, completeness, optimality

✓ Pick the right algorithm

Next lecture: ***informed search*** *(heuristics, A*).*

Next: informed search

Use a **heuristic** $h(n)$ — an estimate of distance to the goal — to bias the search.



A* combines path cost so far ($g(n)$) with heuristic ($h(n)$) — provably optimal under mild conditions.